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Laboratory measurement of strength mobilisation in kaolin: link to stress history

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This letter presents data from triaxial tests conducted as part of a research programme into the stress–strain behaviour of clays and silts at Cambridge University. To support findings from earlier research using databases of soil tests, eighteen CIU triaxial tests on speswhite kaolin were performed to confirm an assumed link between mobilisation strain ($\gamma_{M=2}$) and overconsolidation ratio (OCR). In the moderate shear stress range ($0{\cdot}2c_u$ to $0{\cdot}8c_u$) the test data are essentially linear on log–log plots. Both the slopes and intercepts of these lines are simple functions of OCR.

KEYWORDS: deformation; laboratory tests; plasticity; shear strength; stiffness

NOTATION

A	regression coefficient
b	exponent determined from regression analysis
CIU	consolidated isotropic undrained
c_u	undrained shear strength
c_u	consolidated isotropic undrained
d	exponent determined from regression analysis
e_0	initial void ratio
G	shear modulus
G_0	maximum shear modulus
I_p	plasticity index
M	mobilisation factor c_u/τ_{mob}
m	exponent determined from regression analysis
n	number of data points used to generate a correlation
OCR	overconsolidation ratio
p	the smallest level of significance that would lead to the rejection of the null hypothesis, i.e. that the value of $r = 0$, in the case of determining the p-value for a regression
p_a	atmospheric pressure
p'_c	maximum effective consolidation pressure
p'_0	mean stress in the triaxial after swell back
q	deviator stress
R^2	coefficient of determination of a correlation (the square of the correlation coefficient r)
S	regression coefficient
SE	standard error in a regression, a quantification of deviation about the fitted line
w_L	liquid limit
γ	shear strain, taken as $1{\cdot}5$ times the axial strain (ε_a) in this letter
$\gamma_{M=2}$	mobilisation strain
ε_a	axial strain
κ	slope of unload–reload line
Λ	exponent in the equation of Ladd *et al.* (1977)
λ	slope of normal compression line
σ'_{v0}	vertical effective stress in the ground
σ'_{vc}	maximum past effective vertical stress in the ground
τ_{mob}	mobilised shear stress

INTRODUCTION

Knowledge of soil stiffness and stress–strain behaviour is essential to the calculation of ground displacements that may damage structures. These serviceability considerations, termed SLS in Eurocode 7 (BSI, 2010), should be at the forefront of the geotechnical practitioner's mind. For example, deformations are important in the design of offshore wind turbines, both in terms of dynamic structural response under severe loads and due to the vulnerability of the drive and gearbox to tilting of the mast.

Research has been undertaken at Cambridge University to define and validate simplified mechanistic models in conjunction with soil stress–strain data to enable routine calculations of footing settlements (Osman & Bolton, 2005; Osman *et al.*, 2007) and the displacement of braced retaining structures (Osman & Bolton, 2006; Lam & Bolton, 2011), for example. The calculation procedure is based on conservation of energy and is known as mobilisable strength design (MSD).

An important feature of MSD is the need to model the strength mobilisation of the soil. The shear stiffness of clays and silts at small strains has been shown to be empirically determinable using the maximum shear modulus (G_0) and a quasi-hyperbolic stress–strain relation in which the shear strain required to halve the stiffness was seen to vary with the liquid limit (w_L) (Vardanega & Bolton, 2011a). This letter presents measurements of stress versus strain for kaolin clay for various stress histories and for stress levels approaching failure, in undrained CIU triaxial compression tests. A summary of the basic kaolin parameters from the present study is shown in Table 1.

Previous investigations into the stiffness of reconstituted soils are detailed by Houlsby & Wroth (1991) and Viggiani & Atkinson (1995). Viggiani & Atkinson (1995) fitted equation (1) to laboratory data of the initial shear modulus for a range of clays, including kaolin

$$\frac{G_0}{p_a} = S\left(\frac{p'_0}{p_a}\right)^d \mathrm{OCR}^m \tag{1}$$

where G_0 is the maximum shear modulus, p_a is atmospheric pressure, p'_0 is mean stress (kPa), OCR is the overconsolidation ratio (defined as either p'_c/p'_0 or as $\sigma'_{vc}/\sigma'_{v0}$) and d, m and S are experimentally determined coefficients. Equation (1) shows the degree of overconsolidation to be a key determinant in the prediction of the small-strain stiffness of soils. The relation between degree of overconsolidation and behaviour at larger strain levels is the focus of this letter.

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Table 1. Summary of basic properties of the tested kaolin (numbers in square brackets indicate the number of tests to determine the parameter)

Plastic limit w_P: %	29·6 [4]
Liquid limit w_L: %	62·6 [1]
Slope of normal compression line λ	0·250 [4]
Slope of unload–reload line κ	0·039 [6]

EXPERIMENTAL METHODOLOGY

During triaxial compression, the axial stress is increased while keeping the cell pressure constant. An undrained test maintains constant volume, allowing excess pore pressures to develop. Conventional triaxial testing methodology is outlined in Bishop and Henkel (1957).

In the triaxial tests, an external linear variable differential transformer (LVDT) measures the overall movement of the sample (used to capture the strain data) to an accuracy of 0·125 mm. A strain accuracy of 10^{-3} is sufficient to capture the influence of the OCR on the moderate stress region (defined in the next section), which is the aim of this letter.

Details of the effective stress history of the triaxial tests performed are displayed in Table 2, where p'_c is the maximum effective consolidation pressure, p'_0 is the mean effective stress after swelling back and OCR denotes the ratio between the two. A backpressure of 350 kPa was maintained during the tests.

STRESS–STRAIN BEHAVIOUR

Figure 1 shows the stress–strain curves measured in the eighteen CIU triaxial tests, four of which were reported by Xu (2011). The data are plotted on log–log axes. The range of OCR studied is from 1 to 20. The data are presented in terms of shear strain γ in the triaxial test, which is taken in this letter as

$$\gamma = 1{\cdot}5\varepsilon_a \tag{2}$$

This is done on the premise that (e.g. Terzaghi *et al.*, 1996)

$$\frac{\gamma}{2} = \frac{\varepsilon_1 - \varepsilon_3}{2} \qquad \varepsilon_3 = \frac{-\varepsilon_1}{2} \tag{3}$$

where ε_1 and ε_3 are the principal strains.

Table 2. Stress history of triaxial tests performed

Test ID	p'_c: kPa	p'_0: kPa	OCR
180(1)	180	180	1
180(2)	180	90	2
180(5)	180	36	5
180(10)	180	18	10
180(20)	180	9	20
300(2)	300	150	2
300(5)	300	60	5
300(10)	300	30	10
300(15)	300	20	15
300(20)	300	15	20
500(2)	500	250	2
500(10)	500	50	10
500(15)	500	33·3	15
500(20)	500	25	20
500(1)_Xu	500	500	1
500(2)_Xu	500	250	2
500(5)_Xu	500	100	5
500(10)_Xu	500	50	10

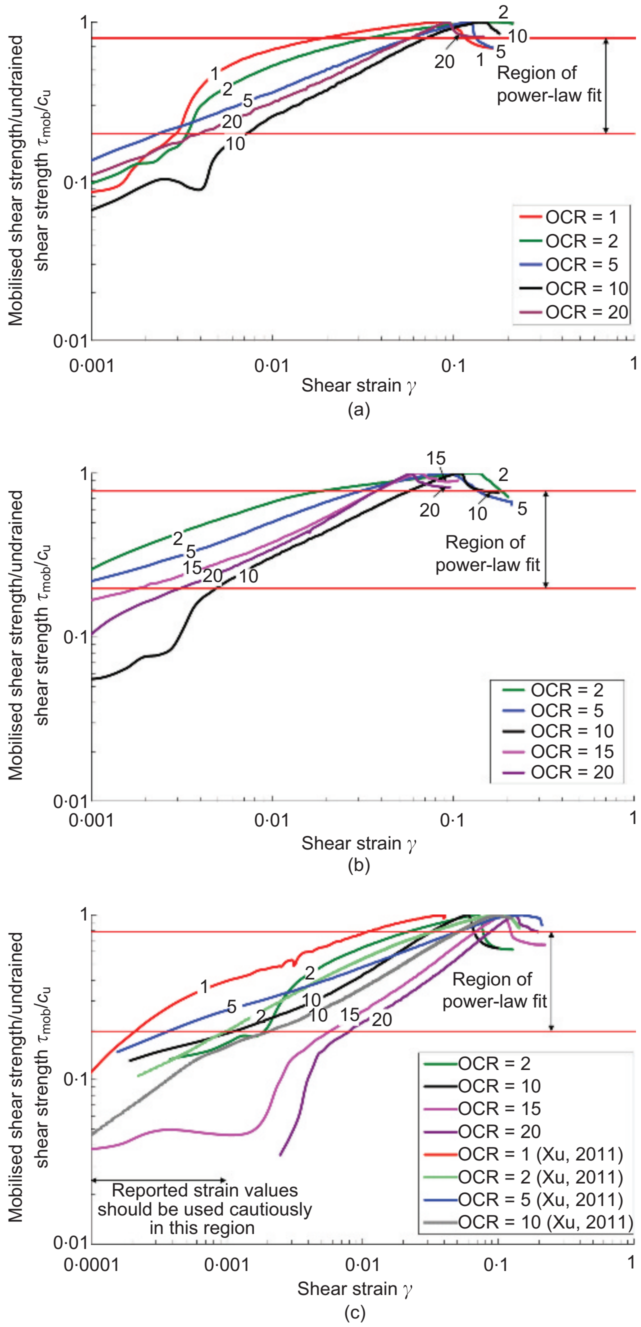

Fig. 1. Triaxial test data for tests with (a) 180 kPa pre-consolidation, (b) 300 kPa pre-consolidation and (c) 500 kPa pre-consolidation

The stress–strain curves from triaxial testing are sensibly linear over a range of moderate stresses when the data are plotted on log–log axes.

Verification of c_u values at varying stress levels

The undrained shear strength ratio $(c_u/\sigma'_{v0})_{nc}$ for normally consolidated soils can be estimated from Skempton's correlation (1954, 1957)

$$(c_u/\sigma'_{v0})_{nc} = 0{\cdot}11 + 0{\cdot}37 I_p \tag{4}$$

From Table 1, the plasticity index I_p for the clay tested in the present study is 0·33. Using equation (4), this gives an empirical value of $(c_u/\sigma'_{v0})_{nc}$ = 0·23, which corresponds reasonably well with the average of the test values of $(c_u/p'_0)_{nc}$ of 0·19 (at p'_c = 500 kPa) and 0·29 (at p'_c = 180 kPa); see Table 3. Equation (5) gives the relationship for

Table 3. Curve-fitting and normalisation parameters

Test ID	A	b	R^2	n	c_u: kPa	e_0	$\gamma_{M=2}$
180(1)	5·861	0·484	0·850	59	51·7	1·15	0·00521
180(2)	3·531	0·425	0·938	79	38·4	1·27	0·00880
180(5)	2·825	0·443	0·999	102	32·0	1·12	0·01969
180(10)	3·882	0·595	1·000	115	19·9	1·29	0·03201
180(20)	3·572	0·530	0·999	139	15·9	1·21	0·02506
300(2)	3·311	0·356	0·980	52	68·6	1·23	0·00445
300(5)	3·034	0·389	0·998	110	45·6	1·17	0·00988
300(10)	3·681	0·540	1·000	140	30·6	1·21	0·02488
300(15)	3·733	0·489	0·991	194	34·2	1·08	0·01737
300(20)	5·082	0·580	0·995	131	27·5	1·19	0·01924
500(2)	4·227	0·423	0·926	50	105·1	1·13	0·00530
500(10)	3·732	0·460	0·991	131	57·5	1·18	0·01337
500(15)	3·750	0·584	0·998	159	36·4	1·14	0·03265
500(20)	3·350	0·602	0·997	200	32·8	1·20	0·04389
500(1)_Xu	3·236	0·311	0·987	45	93·4	n/a	0·00261
500(2)_Xu	2·972	0·377	0·991	96	88·1	n/a	0·00815
500(5)_Xu	1·879	0·291	0·998	96	69·7	n/a	0·01121
500(10)_Xu	3·192	0·470	0·998	133	56·0	n/a	0·01978

overconsolidated soils taken from Ladd *et al.* (1977); in Fig. 2 it is fitted to the triaxial tests data, taking $(c_u/p'_0)_{nc}$ as 0·23. While not identical, the use of c_u/p'_0 and c_u/σ'_{v0} interchangeably in Ladd's equation (as has been done in Fig. 2) was explored by Muir Wood (1990) who concluded that no significant difference results.

$$\frac{c_u/\sigma'_{v0}}{(c_u/\sigma'_{v0})_{nc}} = \mathrm{OCR}^{\Lambda} \tag{5}$$

where σ'_{v0} is the vertical effective stress, OCR is the overconsolidation ratio defined as $\sigma'_{vc}/\sigma'_{v0}$ and Λ is an empirical exponent, which may decrease with increasing OCR (Ladd *et al.*, 1977; Muir Wood, 1990) from 0·85 to 0·75; nc denotes normal consolidation.

From critical state soil mechanics (Schofield and Wroth, 1968), Muir Wood (1990) showed that Λ should be given by

$$\Lambda = \frac{\lambda - \kappa}{\lambda} \tag{6}$$

Based on data collected by Mayne (1980) and presented by Muir Wood (1990), Λ varies between 0·2 and 1·0 with a mean value of 0·63 and a standard deviation of 0·18. This is a significantly greater range than would be implied by equation (5).

Using the values from Table 1 in equation (6), it would be expected that Λ = 0·84, although Muir Wood cautions that it is difficult to determine a reliable value of κ from the mean slope of a swelling line. From Fig. 2, the value of Λ is shown to be 0·68 (when the regression is forced through the origin, as implied by equation (5)), which is slightly lower than the theoretical value; however, it is similar to the mean of previously collected experimental data (Mayne, 1980).

The general form of Ladd's relationship is shown to fit the test data well. This allows one to conclude that the c_u values computed from the test data are not unreasonable.

Mobilisation strain framework

A recent review of strength mobilisation in clays and silts (Vardanega and Bolton, 2011b) presented a large database of 115 tests on natural samples of 19 clays compiled from a range of publications. It was shown that a power law fits the moderate stress region ($0{\cdot}2<\tau_{mob}/c_u<0{\cdot}8$) of the stress–strain curves very well. The power law has the following elementary form (Vardanega & Bolton, 2011b)

$$\tau_{mob}/c_u = A\gamma^b \tag{7}$$

In the previously published database, b for the 115 tests in the database was shown to range from 0·3 to 1·2 with an average of 0·6.

Analysis of new kaolin data

Figure 3 shows power curves fitted to the 18 triaxial tests on reconstituted kaolin. It is observed that power curves do not fit the test data as well at low OCRs as they do at moderate to high OCRs. Figure 4 shows that the b value from equation (7) is related to the OCR via the following regression equation

$$b = 0{\cdot}011(\mathrm{OCR}) + 0{\cdot}371 \tag{8}$$

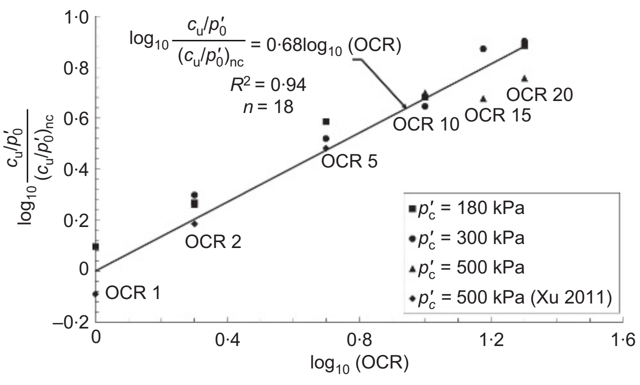

Fig. 2. Fitting the equation of Ladd *et al.* (1977) equation to the test data

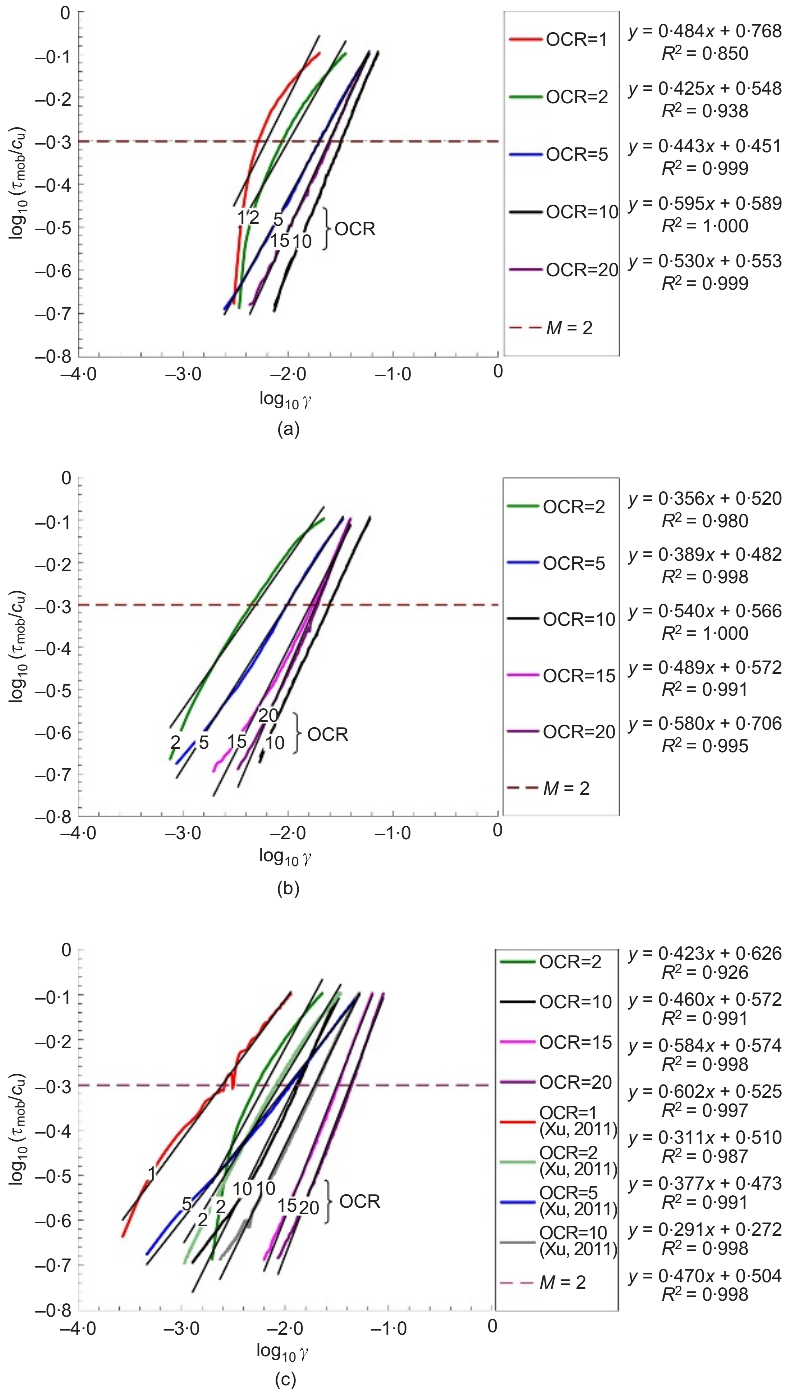

Fig. 3. Fitting power curves to the test data: (a) 180 kPa data (five tests); (b) 300 kPa data (five tests); (c) 500 kPa data (eight tests)

for which $R^2 = 0{\cdot}591$, $n = 18$, SE $= 0{\cdot}064$ and $p < 0{\cdot}001$.

The mobilisation strain is the magnitude of shear strain at which half the shear strength is mobilised (Vardanega & Bolton, 2011b), hence

$$(\gamma_{M=2}) = \left(\frac{0{\cdot}5}{A}\right)^{1/b} \tag{9}$$

Table 3 lists the values of the curve-fitting parameters A and b together with measured values of undrained shear strength c_u and mobilisation strain $\gamma_{M=2}$ for the 18 tests on kaolin. The undrained shear strength is used to normalise the shear stress and the mobilisation strain is used to normalise the shear strain. The resulting prediction equation for shear strength mobilisation has the form

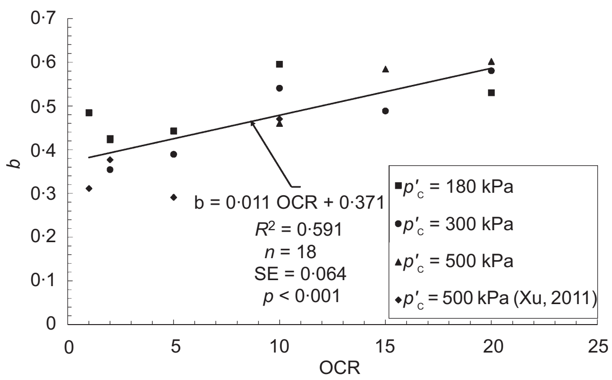

Fig. 4. *b* versus OCR

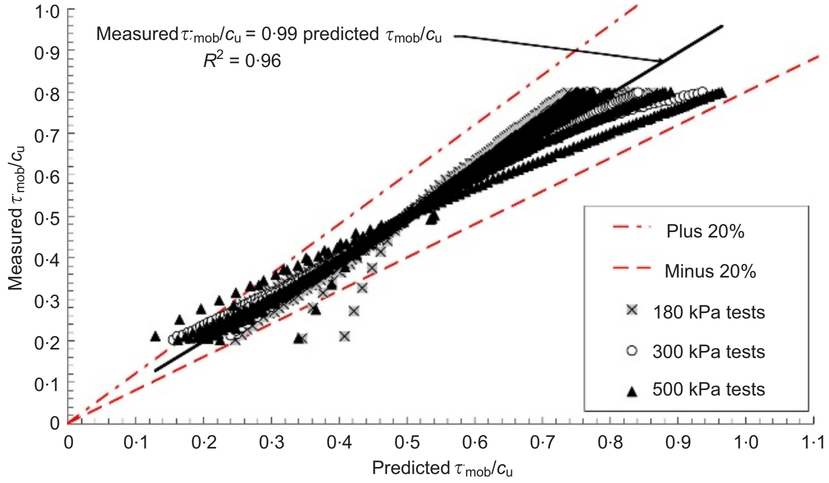

Fig. 5. Predicted versus measured τ_{mob}/c_u using equations (8) and (10)

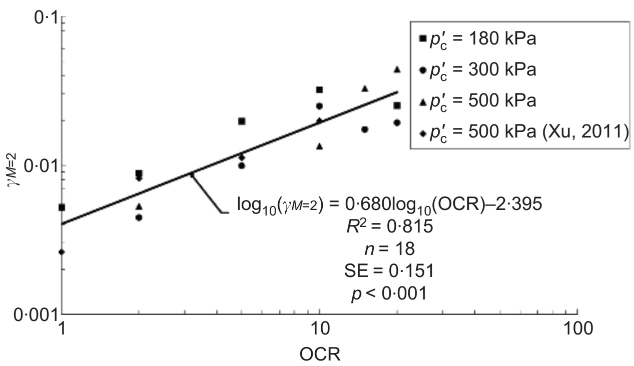

Fig. 6. Logarithm of mobilisation strain versus logarithm of OCR

$$\tau_{mob}/c_u = 0{\cdot}5\left(\frac{\gamma}{\gamma_{M=2}}\right)^b \tag{10}$$

in the range $0{\cdot}2 < \tau_{mob}/c_u < 0{\cdot}8$

Figure 5 shows a plot of τ_{mob}/c_u as predicted using equation (8) for the exponent and equation (10) for the mobilised shear strength ratio, versus corresponding measurements. The R^2 on the plot is 0·96 and the slope is very close to 1·0 (0·99), which validates the use of the two equations in tandem. This level of accuracy is only attained if the mobilisation strain ($\gamma_{M=2}$) is known precisely.

INFLUENCE OF STRESS HISTORY ON MOBILISATION STRAIN

Vardanega & Bolton (2011c) showed that the mobilisation strain is related to depth of sample for a database of tests on natural London clay samples. The observation of $\gamma_{M=2}$ increasing with decreasing depth is akin to suggesting that $\gamma_{M=2}$ increases with OCR. Figure 6 shows the mobilisation strain $\gamma_{M=2}$ plotted against OCR. A good coefficient of determination is observed (R^2 = 0·815) and the p-value is very small (<0·001). For kaolin, the following regression relationship is available

$$\log_{10}(\gamma_{M=2}) = 0{\cdot}680\log_{10}(\mathrm{OCR}) - 2{\cdot}395 \tag{11}$$

for which R^2 = 0·815, n = 18, SE = 0·151 and p < 0·001. Rearranging equation (11) gives

$$(\gamma_{M=2}) = 0{\cdot}0040(\mathrm{OCR})^{0{\cdot}680} \tag{12}$$

Figure 7 shows the predicted versus measured plot when equations (8), (10) and (12) are used to predict τ_{mob}/c_u. The error bands widen to around ±40% due to the scatter about the trend line in Fig. 6. Using equations (8), (10) and

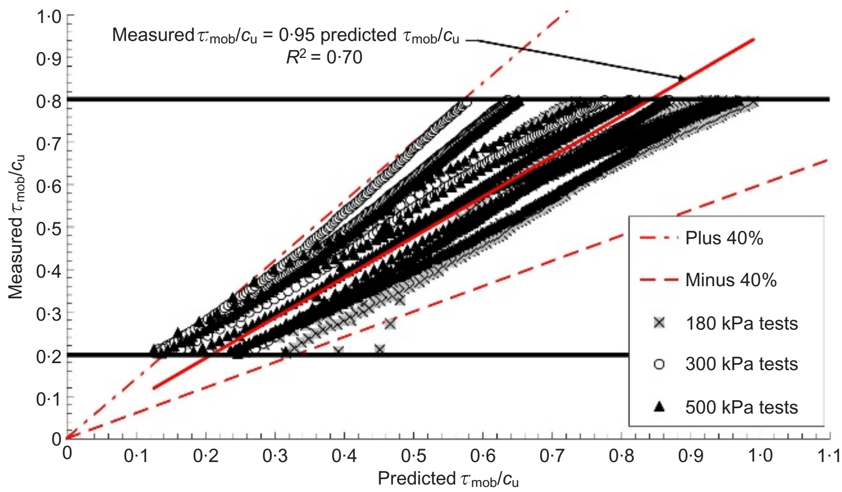

Fig. 7. Measured versus predicted τ_{mob}/c_u using equations (8), (10) and (12)

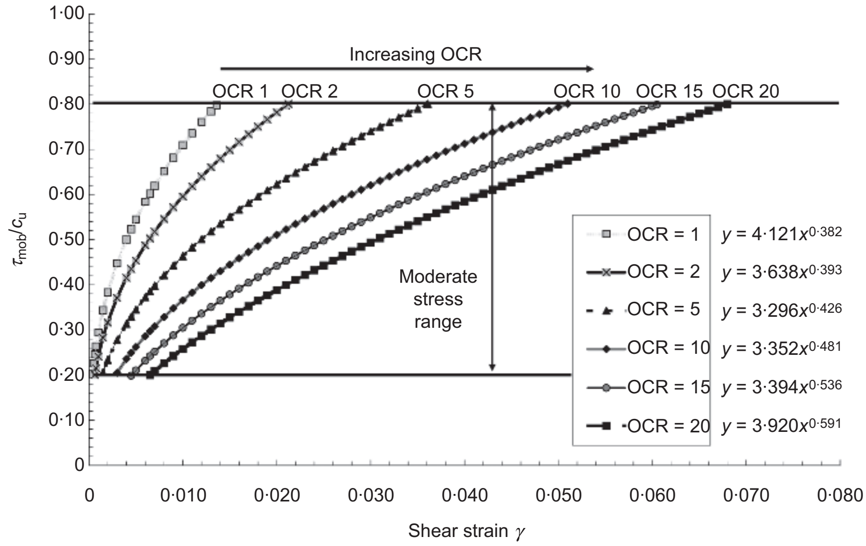

Fig. 8. General stress–strain curves drawn using equations (8), (10) and (12)

(12), the predicted stress–strain curves are drawn in Fig. 8. Similar behaviour is shown in the data of Todi clay presented by Burland *et al*. (1996) and analysed in Vardanega & Bolton (2011b).

Implications for design

The implication for geotechnical design is that less strain is needed to mobilise the same proportion of shear strength the deeper a geotechnical structure is built. For a bored pile in overconsolidated clay, for example, the soil in contact with the shaft at the top of the pile is likely to be significantly more compliant than the soil in contact with the base.

If the pile head settlement at working load is to be calculated using a *t–z* analysis, one must make assumptions about the variation of *t–z* spring behaviour with depth. If a designer assigns a single value of G/c_u for the soil, this implies a single strain to failure at all depths. This letter has shown that such an assumption would be unwarranted.

SUMMARY AND CONCLUSIONS

This letter has focused on establishing a link between mobilisation strain and stress history. The following summary points and conclusions are made.

1. Data from 18 CIU triaxial tests on reconstituted kaolin samples confirm that the stress–strain curves (in the moderate stress region) are roughly linear on log–log plots.
2. The mobilisation strain framework presented by Vardanega & Bolton (2011b, 2011c) is verified for reconstituted kaolin. A simple stress–strain model for kaolin is

$$\tau_{\mathrm{mob}}/c_{\mathrm{u}} = 0{\cdot}5\left(\frac{\gamma}{\gamma_{M=2}}\right)^{b} \text{ in the range } 0{\cdot}2 < \tau_{\mathrm{mob}}/c_{\mathrm{u}} < 0{\cdot}8$$

   where $b = 0{\cdot}011(\mathrm{OCR}) + 0{\cdot}371$. The average exponent b recorded by Vardanega & Bolton (2011b), for natural clays of unknown OCR, was 0·6 within a range of 0·3–1·2. This is not inconsistent with the current data of these tests on reconstituted kaolin.
3. The mobilisation strain $\gamma_{M=2}$ is shown to increase strongly with the logarithm of overconsolidation ratio via the following relationship for kaolin

$$\gamma_{M=2} = 0{\cdot}0040(\mathrm{OCR})^{0{\cdot}680}$$

4. Just as OCR has been previously found (equation (1)) to influence small-strain stiffness, so it has now been demonstrated to influence both the position and slope of stress–strain curves of clay in the region of moderate strength mobilisations, when plotted on log–log axes.

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